

We measure latitude and longitude in degrees but assume that the trigonometric functions used below expect units to be in radians.

Define $\tilde{lat} = \pi \cdot lat/180$, and

$$\tilde{Y} = \frac{1}{2\pi} \log \left(\frac{1 + \sin(\tilde{lat})}{1 - \sin(\tilde{lat})} \right)$$

then , with $Y = (2^{zoom}) * ((1 - \tilde{Y})/2) \Rightarrow \tilde{Y} = 1 - Y/2^{zoom-1}$

$$\tilde{lat} = 2\pi n \pm \sin^{-1} \left(\frac{\exp 2\pi \tilde{Y} - 1}{\exp 2\pi \tilde{Y} + 1} \right) + \pi, n \in \mathbb{Z}$$

For longitude, the inverse mapping is much simpler:

$$X = (2^{zoom}) * ((\tilde{X} + 1)/2) \Rightarrow \tilde{X} = X/2^{zoom-1} - 1$$

Since $\tilde{X} = lon/180$, we get

$$lon = 180 \cdot (X/2^{zoom-1} - 1) .$$