

# Package ‘DiscreteLaplace’

January 29, 2012

**Type** Package

**Title** Discrete Laplace distribution

**Version** 1.0

**Date** 2012-01-29

**Author** Alessandro Barbiero <alessandro.barbiero@unimi.it>, Riccardo  
Inchingolo <riccardo.arner@libero.it>

**Maintainer** Alessandro Barbiero <alessandro.barbiero@unimi.it>

**Description** Density, distribution function, quantile function, random  
generation and estimation for the skew discrete Laplace distribution

**License** GPL

**LazyLoad** yes

## R topics documented:

DiscreteLaplace-package . . . . .	1
ddlplace . . . . .	2
Edlplace . . . . .	3
estdlplace . . . . .	4
pdlplace . . . . .	5
qdlplace . . . . .	6
rdlplace . . . . .	7
<b>Index</b>	<b>9</b>

---

DiscreteLaplace-package  
*Discrete Laplace distribution*

---

## Description

Density, distribution function, quantile function and random generation for the skew discrete Laplace distribution

## Details

Package: DiscreteLaplace  
 Type: Package  
 Version: 1.0  
 Date: 2012-01-29  
 License: GPL  
 LazyLoad: yes

### Author(s)

Alessandro Barbiero, Riccardo Inchingolo Maintainer: Alessandro Barbiero <alessandro.barbiero@unimi.it>

### References

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AIMS*, 58: 555-571

---

ddlaplace

*Density function of the discrete Laplace distribution*


---

### Description

The function computes the density function of the discrete Laplace distribution

### Usage

```
ddlaplace(x, p, q)
```

### Arguments

x	a vector of integer values
p	the first parameter of the discrete Laplace distribution
q	the second parameter of the discrete Laplace distribution

### Details

$$P(X = x; p, q) = \frac{(1-p)(1-q)}{1-pq} p^x; x = 0, 1, 2, 3, \dots$$

$$P(X = x; p, q) = \frac{(1-p)(1-q)}{1-pq} q^{|x|}; x = 0, -1, -2, -3, \dots$$

### Value

The vector of probabilities corresponding to the vector of integer values x

### Author(s)

Alessandro Barbiero, Riccardo Inchingolo

## References

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AISM*, 58: 555-571

## See Also

[pdlaplace](#), [qdlaplace](#), [rdlaplace](#)

## Examples

```
p<-0.7
q<-0.45
x<--10:10
prob<-ddlplace(x, p, q)
plot(x, prob, type="h")
# swap the parameters
prob<-ddlplace(x, q, p)
plot(x, prob, type="h")
```

---

Edlplace

---

*Moments of the discrete Laplace distribution*


---

## Description

The function provides the expected value and the variance of the discrete Laplace distribution, and the expectation of its absolute value

## Usage

```
Edlplace(p, q)
```

## Arguments

p	the first parameter of the discrete Laplace distribution
q	the second parameter of the discrete Laplace distribution

## Details

$$E(X; p, q) = \frac{1}{1-p} - \frac{1}{1-q} = \frac{p}{1-p} - \frac{q}{1-q},$$

$$E(|X|; p, q) = \frac{q(1-p)^2 + p(1-q)^2}{(1-qp)(1-q)(1-p)},$$

$$V(X; p, q) = \frac{1}{(1-p)^2(1-q)^2} \left[ \frac{q(1-p)^3(1+q) + p(1-q)^3(1+p)}{1-pq} - (p-q)^2 \right]$$

## Value

A list of three items:

E1	expected value
E1a	expectation of the absolute value
V	variance

**Author(s)**

Alessandro Barbiero, Riccardo Inchingolo

**References**

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AISM*, 58: 555-571

**See Also**

[ddlplace](#)

**Examples**

```
# ex.1
p<-0.5
q<-0.4
Edlaplace(p, q)
# ex.2
p<-0.1
q<-0.9
Edlaplace(p, q)
```

---

estdlaplace

*Sample estimation for the discrete Laplace distribution*

---

**Description**

The function provides the maximum likelihood estimates for the parameters of the discrete Laplace distribution and the estimate of the inverse of the Fisher information matrix. The method of moments estimators of  $p$  and  $q$  coincide with the maximum likelihood's.

**Usage**

```
estdlaplace(x)
```

**Arguments**

$x$  a vector of integer values sampled from the discrete Laplace distribution

**Details**

See the reference. If  $\bar{x}^+ = \frac{1}{n} \sum_{i=1}^n x_i^+$ ,  $\bar{x}^- = \frac{1}{n} \sum_{i=1}^n x_i^-$  where  $x^+$  and  $x^-$  are the positive and the negative parts of  $x$ , respectively:  $x^+ = x$  if  $x \geq 0$  and zero otherwise,  $x^- = (-x)^+$ , then

$$\hat{q} = \frac{2\bar{x}^-(1+\bar{x})}{1+2\bar{x}^-\bar{x}+\sqrt{1+4\bar{x}^-\bar{x}^+}}, \hat{p} = \frac{\hat{q}+\bar{x}(1-\hat{q})}{1+\bar{x}(1-\hat{q})}$$

when  $\bar{x} \geq 0$  and

$$\hat{p} = \frac{2\bar{x}^+(1-\bar{x})}{1-2\bar{x}^+\bar{x}+\sqrt{1+4\bar{x}^-\bar{x}^+}}, \hat{q} = \frac{\hat{p}-\bar{x}(1-\hat{p})}{1-\bar{x}(1-\hat{p})}$$

when  $\bar{x} \leq 0$ .

**Value**

A list comprising

hatp	estimate of $p$
hatq	estimate of $q$
hatSigma	estimate of the inverse of the Fisher information matrix

**Author(s)**

Alessandro Barbiero, Riccardo Inchingolo

**References**

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AIMS*, 58: 555-571

**See Also**

[ddlaplace](#), [pdlaplace](#), [qdlaplace](#), [rdlaplace](#)

**Examples**

```
p<-0.6
q<-0.3
n<-20
x<-rdlaplace(n, p, q)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
# increase n
n<-100
x<-rdlaplace(n, p, q)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
# swap the parameters
x<-rdlaplace(n, q, p)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
```

---

pdlaplace

*Distribution function of the discrete Laplace distribution*

---

**Description**

The function provides the cumulate probabilities associated to the values of a given vector of real values

**Usage**

```
pdlaplace(x, p, q)
```

**Arguments**

x	a vector of real values
p	the first parameter of the discrete Laplace distribution
q	the second parameter of the discrete Laplace distribution

**Details**

$$F(x; p, q) = P(X \leq x) = \frac{(1-p)q^{-[x]}}{1-pq}, x < 0$$

$$F(x; p, q) = P(X \leq x) = 1 - \frac{(1-q)p^{[x]+1}}{1-pq}, x \geq 0$$

**Value**

A vector of cumulate probabilities

**Author(s)**

Alessandro Barbiero, Riccardo Inchingolo

**References**

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AIMS*, 58: 555-571

**See Also**

[ddlplace](#), [qdlplace](#), [rdlplace](#)

**Examples**

```
p<-0.2
q<-0.5
x<-c(-3, -1, pi)
pdlaplace(x, p, q)
```

---

qdlaplace

*Quantile function of the discrete Laplace distribution*

---

**Description**

The function provides the values corresponding to a vector of desired cumulate probabilities

**Usage**

```
qdlaplace(prob, p, q)
```

**Arguments**

prob	a vector of cumulate probabilities
p	the first parameter of the discrete Laplace distribution
q	the second parameter of the discrete Laplace distribution

**Value**

A vector of integer values

**Author(s)**

Alessandro barbiero, Riccardo Inchingolo

**References**

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AIMS*, 58: 555-571

**See Also**

[ddlaplace](#), [pdlaplace](#), [rdlaplace](#)

**Examples**

```
p<-0.8
q<-0.4
prob<-c(0.2,0.5,0.8)
x<-qdlaplace(prob, p, q)
x
# check
upper<-pdlaplace(x, p, q)
upper
lower<-pdlaplace(x-1, p, q)
lower
lower<=prob & prob<=upper
```

---

rdlaplace

*Random generation for the discrete Laplace distribution*


---

**Description**

The function provides a random sample of size  $n$  from the discrete Laplace distribution

**Usage**

```
rdlaplace(n, p, q)
```

**Arguments**

n	the sample size
p	the first parameter of the discrete Laplace distribution
q	the second parameter of the discrete Laplace distribution

**Value**

A vector of size of integer values  $n$

**Author(s)**

Alessandro Barbiero, Riccardo Inchingolo

**References**

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *AIISM*, 58: 555-571

**See Also**

[ddlaplace](#), [pdlaplace](#), [qdlaplace](#)

**Examples**

```
n<-100
p<-0.3
q<-0.5
x<-rdlaplace(n, p, q)
x
t<-table(x)
t
plot(t)
```



# Index

## \*Topic **distribution**

- ddlplace, [2](#)
- Edlplace, [3](#)
- estdlplace, [4](#)
- pdlplace, [5](#)
- qdlplace, [6](#)
- rdlplace, [7](#)

## \*Topic **package**

- DiscreteLaplace-package, [1](#)

ddlplace, [2](#), [4–8](#)

DiscreteLaplace  
    (*DiscreteLaplace-package*),  
    [1](#)

DiscreteLaplace-package, [1](#)

Edlplace, [3](#)

estdlplace, [4](#)

pdlplace, [3](#), [5](#), [5](#), [7](#), [8](#)

qdlplace, [3](#), [5](#), [6](#), [6](#), [8](#)

rdlplace, [3](#), [5](#), [6](#), [7](#), [7](#)